Inductive instability in heterogeneous nonstationary systems

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In this study we analyze a new type of electric dynamo caused by the rapid change of the distribution of the electric conductivity in heterogeneous conducting systems. It is demonstrated that there exist two types of electric dynamos, namely, the regular magnetic dynamo and the electric current dynamo. The magnetic dynamo is associated with the growth of the total energy of the magnetic field. The electric current dynamo is defined as the growth of the total electric current through some cross section of a conductor, whereby the choice of the cross section is determined by the symmetry of the excited electromagnetic field. We show that the condition for the excitation of the electric current dynamo is less restrictive than the condition for the excitation of the magnetic dynamo, and it can be satisfied even without a hydrodynamic flow. The existence of the hydrodynamic flow is cardinal for the excitation of the magnetic dynamo. In contrast to the turbulent magnetic dynamo which is associated with the fact that magnetic-field lines are ''frozen in'' to the fluid and thus can be excited at high magnetic Reynolds numbers, the laminar magnetic dynamo which is considered in the present study can be excited at the relatively low magnetic Reynolds numbers $Re_m \geq 1$ depending upon the symmetry of the electromagnetic field. In this study we determined the dependence of the magnetic Reynolds number providing the excitation of the instability upon the symmetry of the electromagnetic field. [S1063-651X(97)03209-1]

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I. INTRODUCTION

In our previous study [1] we showed that the motion of a jump of an electric conductivity with a sufficiently high velocity of the front causes a transition of the system into the unstable with respect to the spontaneous excitation of the electromagnetic field state. The mechanism of the instability is associated with the work performed by the source of motion, which depends upon the direction of the interface velocity and the direction of the ponderomotive forces. In a case when the ponderomotive forces impede the motion of the interface the energy from the source of motion is transformed into the energy of the electromagnetic field. When such a process occurs sufficiently fast, the rate of energy dissipation is not enough to compensate for the growth of the electromagnetic field caused by the work of the ponderomotive forces.

In our previous study [1] we investigated the instability of the infinitely long cylindrical conductor with a moving boundary with respect to the spontaneous excitation of the electric current. This model of an infinitely long conductor restricts the range of application of the obtained results and their theoretical foundation. Indeed, in an infinitely long conductor the effect may be overestimated since the electromagnetic field (vector potential) is logarithmically divergent far from the conductor's surface. Therefore, from the point of view of various applications and for the theoretical validation of the instability, it is desirable to analyze this problem for a finite system. Such a problem is considered in the present study for a system with a spherical symmetry.

The investigated instability is accompanied by a redistribution of the magnetic field, from regions far away from the conductor to regions where the electric current density is essentially nonzero. The situation is similar to the self-focusing in the nonlinear optical wave [2], although this similarity is only of a formal character since the involved physical mechanisms are completely different.

Apart from the inductive instability occurring in a moving medium, we consider the feasibility of the occurrence of the electric current dynamo in a medium without hydrodynamic flow. It is shown that such instability can be caused by a rapid decrease of the inductance. Such a rapid decrease of inductance in the heterogeneous conductors can occur not necessarily due to a hydrodynamic flow, but can be caused by a variation of the electric resistance of some regions in the conductor or by variation of its geometry.

The possibility for a transition of the system into a state with a negative damping resistance, i.e., an electric current dynamo, was discussed first in Ref. [3]. In Ref. [4] we determined the magnitude of the threshold electric current I^*, whereby the velocity of motion of the electric conductivity jump becomes sufficient for the transition of the system into a state with a negative damping resistance caused by the rapid decrease of the inductance. In Ref. [5] we analyzed the electric current instability in layered conductors caused by rapid variation of the resistance of the layers, and considered electric current instability in an electric circuit with conductors connected in parallel.

The main goal of this study is to investigate electric current and magnetic instabilities in systems with a spherical symmetry. The analysis of this problem is rather simple, and can be of interest in astrophysics and geophysics.

The paper is organized as follows. In Sec. II we present a general analysis of the magnetic and electric current insta-

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bilities using Maxwell equations. In Sec. III we study magnetic and electric current dynamos in a conducting imploding sphere, and determine the structure of the excited fields depending upon their symmetry. In Sec. IV we discuss a possibility for the excitation of the electric current dynamo due to rapid variation of the spatial distribution of electric conductivity in a medium without hydrodynamic flow.

II. GENERAL CONDITIONS FOR EXCITATION OF ELECTRIC AND MAGNETIC DYNAMOS IN A CONDUCTING MEDIUM

A general condition for excitation of the electromagnetic field in a conducting medium can be determined from the energy balance equation, which can be derived directly from Maxwell equation [6,7]

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{H}}{\partial t}, \quad \vec{\nabla}\times\vec{H}=\frac{4\pi}{c}\vec{j}, \quad \frac{\vec{j}}{\sigma}=\vec{E}+\frac{1}{c}(\vec{v}\times\vec{H}),$$
(1)

where $\vec{E}(\vec{r},t)$ and $\vec{H}(\vec{r},t)$ are electric and magnetic fields, respectively, $\vec{j}(\vec{r},t)$ is a density of an electric current, $\sigma(\vec{r},t)$ is an electric conductivity, and $\vec{v}(\vec{r},t)$ is the velocity of the medium. Equations (1) yield the following equation for the energy balance [5]:

$$\frac{1}{8\pi}\int\frac{\partial}{\partial t}\vec{H}^2 dV=-\int\frac{\vec{j}^2}{\sigma}dV-\frac{1}{c}\int\vec{v}\cdot(\vec{j}\times\vec{H})dV.$$
(2)

According to Eq. (2) the total rate of change of the energy of electromagnetic field is equal to the sum of Joule energy dissipation and the work performed to sustain fluid flow in the field of the ponderomotive forces [the last term in Eq. (2)]. If this work is performed against the ponderomotive forces and it is greater that the dissipation rate, then

$$\frac{\partial}{\partial t}\int\vec{H}^2 dV>0,$$
(3)

i.e., there occurs a magnetic dynamo.

Equation (2) shows that, in a medium without hydrodynamic flow [$\vec{v}(\vec{r},t)=0$], the magnetic dynamo defined by Eq. (3) does not occur. However, as it was demonstrated in Ref. [5], these conditions do not prevent from the occurrence of the electric current dynamo whereby

$$\frac{\partial}{\partial t}(I^2)\geq 0,$$
(4)

where $I(t)$ is the total electric current through the cross section of a conductor. The choice of the cross section depends upon a symmetry of the problem and the distribution of the electric current $\vec{j}(\vec{r},t)$. Thus, e.g., in the case of a conducting sphere with a radius $\bar{r}(t)$ with excited azimuthal electric current $\vec{j}=(0,0,j_\varphi)$,

$$I(t)=\int_0^\pi\int_0^{\bar{r}(t)}j_\varphi d\theta\, r\, dr$$
(5)

Inequality (4) is valid when the following condition is satisfied:

$$R+\dot{L}<0,$$
(6)

where an Ohmic resistance $R(t)$ and inductance $L(t)$ are defined as follows:

$$I^2 R(t)=\int\frac{j^2}{\sigma}dV, \quad \frac{LI^2}{2}=\int\frac{\vec{H}^2}{8\pi}dV.$$
(7)

Indeed, direct substitution of Eqs. (7) into Eq. (2) shows that inequality (6) is a necessary and sufficient condition for the occurrence of the electric current dynamo (4). In heterogeneous conductors the inductance may vary even without a flow of a conducting fluid due to a rapid change of the local conductivity [5].

In Sec. III we consider a magnetic dynamo in an imploding sphere which is similar to the dynamo analyzed in Ref. [3]. This dynamo is different from the turbulent magnetic dynamo [6], and is caused by the motion of the external surface of the conducting fluid. Thus the fluid velocity is directed along the gradient of the electric conductivity. The latter renders the Cowling theorem (see, e.g., Ref. [6]) invalid, and this is the reason that such a dynamo does not require a complex geometry of a magnetic field and can occur at relatively low values of magnetic Reynolds numbers $\mathrm{Re}_m\geq 1$.

III. MAGNETIC DYNAMO IN A PROBLEM WITH A SPHERICAL GEOMETRY

Consider a case with a spherical symmetry with a toroidal vector potential (see, e.g., Refs. [6] and [7]) $\vec{A}=(0,0,A_\varphi)$, which determines electric and magnetic fields:

$$\vec{E}=-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \quad \vec{H}=\vec{\nabla}\times\vec{A}.$$
(8)

Then using Eq. (1), we arrive at the equation for the vector potential:

$$\vec{\nabla}^2\vec{A}=\frac{4\pi\sigma}{c^2}\left(\frac{\partial\vec{A}}{\partial t}-\vec{v}\times\vec{\nabla}\times\vec{A}\right).$$
(9)

We seek a solution for the asimutal component of a vector potential $A_\varphi(r,t)$ in the following self-similar form:

$$A_\varphi(r,t)=a(t)b(\theta)\Phi(\chi), \quad 0<\theta<\pi,$$
(10)

where $\chi=r/\bar{r}(t)$ and $\bar{r}(t)$ is a characteristic dimensional time-dependent parameter of the problem; the physical meaning of $\bar{r}(t)$ will be elucidated below.

In order to provide the toroidal and self-similar field $A_\varphi(r,t)$ we will assume that $\vec{v}(r,t)$ can be presented as

$$\vec{v}(r,t)=\vec{n}u(\chi)\dot{r}, \quad a(t)=\left(\frac{\bar{r}(t)}{r_0}\right)^\lambda,$$
(11)

where $\vec{n}$ is a unit vector normal to the surface, $r_0=\bar{r}(0)$, and $u(\chi)$ is a dimensionless function. Then, using Eqs. (9)–(11), we obtain the following equations for $b(\theta)$ and $\Phi(\chi)$:

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial b}{\partial\theta}\right)-\frac{b(\theta)}{\sin^2(\theta)}=-\ell(\ell+1)b(\theta),\quad(12)$$

$$\frac{\partial^2\Phi}{\partial\chi^2}+\frac{2}{\chi}\frac{\partial\Phi}{\partial\chi}-\frac{\ell(\ell+1)}{\chi^2}\Phi=\nu(\chi)S(\chi),\qquad(13)$$

where

$$S(\chi)=\Phi(\chi)\left(\lambda+\frac{u(\chi)}{\chi}\right)+\frac{\partial\Phi}{\partial\chi}(u(\chi)-\chi),\tag{14}$$

$$\nu(\chi)=\frac{4\pi}{c^2}\sigma(\chi)\dot{\overline{r}}(t)\overline{r}(t).$$

In order to provide the self-similarity of the problem, the electric conductivity $\sigma(\chi)$ must satisfy the condition $\sigma(\chi)=\sigma_0(t)\overline{\sigma}(\chi)$, where $\overline{\sigma}(\chi)$ is a dimensionless function, and

$$\frac{4\pi}{c^2}\sigma_0(t)\dot{\overline{r}}(t)\overline{r}(t)=\nu_0,\quad\nu_0=\mathrm{const}.\tag{15}$$

Solutions of Eq. (12) without singularity in the whole range $0<\theta<\pi$ are associated Legendre polynomials (see, e.g., Ref. [8]), $b(\theta)=P_\ell^1(\cos\theta)$, $\ell=1,2,\dots$.

Before turning to the analysis of some specific models for $\overline{\sigma}(\chi)$ and $u(\chi)$, let us derive first some relations which are required for the analysis of the instabilities. Using the last of Eqs. (1) for the electric current density, and Eqs. (8) and (10), we arrive at the formulas for the asimutal component of an electric current density j_φ, normal component of magnetic field H_n and component H_θ:

$$j=-\frac{ca(t)}{4\pi\overline{r}^2(t)}\nu_0\overline{\sigma}(\chi)P_\ell^1(\mu)S(\chi),$$

$$H_\theta=\frac{a(t)}{\overline{r}(t)}P_\ell^1(\mu)\left(\frac{\partial\Phi}{\partial\chi}+\frac{\Phi}{\chi}\right),\tag{16}$$

$$H_n=\frac{a(t)}{\overline{r}(t)}P_\ell(\mu)\ell(\ell+1)\frac{\Phi(\chi)}{\chi},$$

where $\mu=\cos\theta$ and $P_\ell(\mu)\equiv P_\ell^0(\mu)$.

Then the energy of a magnetic field $W_m=\int(\vec{H}^2/8\pi)dV$ is determined by the following expression:

$$W_m=\frac{1}{2}\frac{a^2(t)\overline{r}(t)\ell(\ell+1)}{2\ell+1}[I_1+\ell(\ell+1)I_2],\tag{17}$$

where

$$I_1=\int_0^\infty\left(\frac{\partial\Phi}{\partial\chi}+\frac{\Phi}{\chi}\right)^2\chi^2d\chi,\quad I_2=\int_0^\infty\Phi^2(\chi)d\chi,\tag{18}$$

Then using the known formulas for the associated Legendre polynomials (see, e.g., Ref. [8]):

$$\int_{-1}^1[P_\ell(\mu)]^2d\mu=\frac{2}{2\ell+1},$$

$$\int_{-1}^1P_\ell^1(\mu)\frac{d\mu}{\sqrt{1-\mu^2}}=1-(-1)^\ell,$$

$$\int_{-1}^1[P_\ell^1(\mu)]^2d\mu=\frac{2\ell(\ell+1)}{2\ell+1},$$

we determine the total current $I(t)$ for odd values of ℓ from Eqs. (5) and (16):

$$I(t)=-\frac{\nu_0ca(t)}{2\pi}B_1,\quad B_1=\int_0^\infty\chi S(\chi)\overline{\sigma}d\chi.\tag{19}$$

According to the last equation and Eqs. (7), the formula for inductance L reads

$$L=\frac{\overline{r}(t)}{c^2}\frac{4\pi^2\ell(\ell+1)}{2\ell+1}\frac{1}{\nu_0^2}\frac{1}{B_1^2}[I_1+\ell(\ell+1)I_2],\quad(20)$$

Using the first of Eqs. (7) and the derived above expressions for $j(\vec{r},t)$ and $I(t)$, we arrive at the formula for Ohmic resistance $R(t)$:

$$R(t)=\frac{\pi}{2}\frac{B_0}{B_1^2}\frac{1}{\sigma_0\overline{r}(t)},\tag{21}$$

where $B_0=\int_0^\infty\overline{\sigma}\chi^2S^2(\chi)d\chi$.

Then condition (6) can be written as

$$\nu_0<-\frac{\ell(\ell+1)}{2\ell+1}\frac{I_1+\ell(\ell+1)I_2}{B_0}.\tag{22}$$

Inequality (22) determines the threshold value of the parameter ν_0 and the minimum absolute value of the spatial scale variation $\dot{\overline{r}}(t)$ which ensure the validity of condition (6). Note that the time variation of a spatial scale does not necessarily require a hydrodynamic flow and can occur due to propagation of a front-separating regions with different electric conductivities, e.g., during melting.

It is worthwhile noting that, according to Eq. (20), an inductance decreases when $\dot{\overline{r}}(t)<0$. Here lies the principal difference of the case with a spherical symmetry from the case with a cylindrical symmetry which we analyzed in Ref. [1], and where inductance decreases if the characteristic length scale $\overline{r}(t)$ growth.

Now we turn to the analysis of the situation with some particular functions $\overline{\sigma}(\chi)$ and $u(\chi)$. Consider a homogeneous sphere with a radius $\overline{r}(t)$ with uniform density $\gamma(t)$ and conductivity $\sigma(t)$ embedded in a dielectric, i.e., for $r<\overline{r}(t)$, and $\sigma=0$ and $\gamma(t)=0$ for $r>\overline{r}(t)$. In this case a continuity equation $\dot{\gamma}+\vec{\nabla}\cdot(\gamma\vec{v})=0$ yields $u(\chi)=\chi$ and

$$S(\chi)=(\lambda+1)\Phi(\chi),\quad\nu(\chi)=\nu_0.\tag{23}$$

Nonsingular solutions of Eq. (13) with functions (23) in the domain $0<\chi<1$ with a nonzero electric current read

ℓ	1	2	3	4	5
k	3.14	4.49	5.96	6.98	8.18

$$\Phi(\chi) = \frac{J_{\ell+1/2}(k\chi)}{\sqrt{\chi}}, \qquad (24)$$

where $J_{\ell+1/2}(z)$ is the Bessel function and $k^2 = -\nu_0(\lambda+1)$.

In the domain $\chi > 1$, electric current vanishes, and a condition of the continuity of a vector potential yields

$$\Phi(\chi) = \frac{\Phi(1)}{\chi^{\ell+1}}, \qquad \Phi(1) = J_{\ell+1/2}(k). \qquad (25)$$

Using the conditions of continuity of magnetic field and Eqs. (24) and (25), we find that

$$\frac{\partial}{\partial\chi}\left(\frac{J_{\ell+1/2}(k\chi)}{\sqrt{\chi}}\right)\bigg|_{\chi=1} = -(\ell+1)J_{\ell+1/2}(k). \qquad (26)$$

Using the identity for functions $J_\lambda(z)$,

$$\frac{\partial J_\lambda(z)}{\partial z} = \frac{\lambda}{z}J_\lambda(z) - J_{\lambda+1}(z),$$

Eq. (26) can be written as

$$(2\ell+1)J_{\ell+1/2}(k) = kJ_{\ell+3/2}(k). \qquad (27)$$

In the region $\nu_0(\lambda+1) > 0$, $k = i\alpha$ and $\alpha > 0$, and Eq. (27) can be rewritten as follows:

$$(2\ell+1)I_{\ell+1/2}(\alpha) = -\alpha I_{\ell+3/2}(\alpha), \qquad (28)$$

where $I_\lambda(\alpha)$ are modified Bessel functions of a real argument. In the range $\alpha > 0$, $I_\lambda(\alpha) > 0$, Eq. (28) does not have roots.

Thus Eq. (27) has roots in the region $\nu_0(\lambda+1) < 0$. Let $k(\ell) > 0$ be the roots of Eq. (27). Then $\nu_0(\lambda+1) = -k^2(\ell)$ or

$$\lambda = -\frac{k^2(\ell)}{\nu_0} - 1. \qquad (29)$$

Using Eqs. (11) and (17) we find that $\dot{W}_m = (2\lambda+1)W_m(t)\dot{\bar{r}}(t)/\bar{r}(t)$, i.e.,

$$W_m(t) = W_0\left(\frac{\bar{r}(t)}{\bar{r}_0}\right)^{2\lambda+1}. \qquad (30)$$

In the region $\nu_0 > 0$ $[\dot{\bar{r}}(t) > 0]$, according to Eq. (29) $\lambda < -1$, and Eq. (30) yields $\dot{W}_m(t) < 0$. Thus, in a case of the expanding sphere a magnetic dynamo does not occur. Since in this case an inductance increases, the electric current dynamo is not excited either.

In the range $\nu_0 < 0$ condition $\dot{W}_m(t) > 0$ implies that $\lambda < -1/2$. Then, according to Eq. (29),
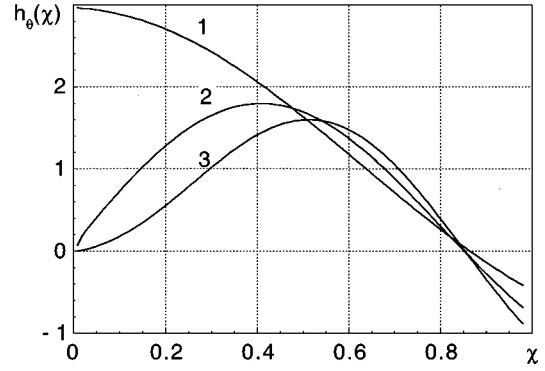

FIG. 1. Tangential component of magnetic field h_θ vs χ for different angular momenta ℓ (1, $\ell = 1$; 2, $\ell = 3$; 3, $\ell = 5$).

$$\nu_0 < -2k^2(\ell). \qquad (31)$$

Using Eq. (19) for the total electric current we find that

$$\frac{\partial I^2}{\partial t} = 2\lambda I^2 \frac{\dot{\bar{r}}(t)}{\bar{r}(t)}, \qquad I^2(t) = I^2(0)\left(\frac{\bar{r}(t)}{\bar{r}_0}\right)^{2\lambda}. $$

A condition for excitation of an electric current dynamo yields $\nu_0 < 0$, and $\lambda < 0$, which, according to Eq. (29), can be satisfied in the range

$$\nu_0 < -k^2(\ell). \qquad (32)$$

In the range of velocities $-2k^2(\ell) < \nu_0 < -k^2(\ell)$ the electric current dynamo is excited but a magnetic dynamo does not occur.

In Table I we show values $k(\ell)$ which are the solutions of Eq. (27). As can be seen from this table the threshold velocity which is required for the transition of the system into a regime of generation increases with the increase of the angular momentum ℓ. In Figs. 1 and 2 we show the variation of magnetic fields $h_\theta(\chi) = [\partial\Phi(\chi)/\partial\chi] + [\Phi(\chi)/\chi]$ and $h_n(\chi) = [\Phi(\chi)/\chi]$ vs χ for different ℓ. As can be seen from these figures, the maximum of the magnetic field occurs at $\chi < 1$, and for $\bar{r}(t) \to 0$ the domain with a maximum of magnetic field implodes into a region with small r.
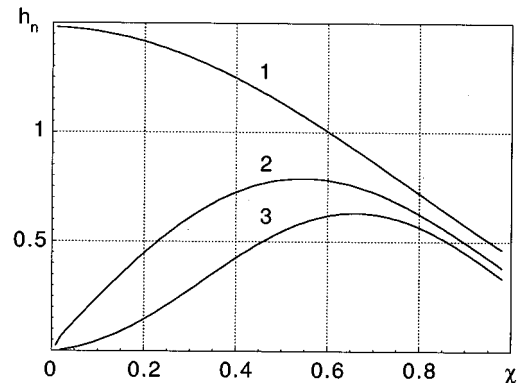

FIG. 2. Normal component of magnetic field h_n vs χ for different angular momenta ℓ (1, $\ell = 1$; 2, $\ell = 3$; 3, $\ell = 5$).

Note that in the basis of functions $\Phi(\chi) \equiv \Phi(k\chi)$ it is possible to construct the general solution for the wide class of initial conditions. In this study we analyzed only the spectra in the system, and have not addressed this problem.

IV. ABOUT THE FEASIBILITY OF AN ELECTRIC CURRENT DYNAMO IN A MEDIUM WITHOUT HYDRODYNAMIC FLOW

It was shown above that when condition (6) is satisfied the electric current dynamo can occur in the system even without hydrodynamic flow, i.e., with $u(\chi) = 0$. However there arises a question whether it is possible to satisfy a condition $R + \dot{L} < 0$ in real systems at least "kinematically." This question can be formulated differently, namely, whether an equation of magnetic diffusion, boundary conditions of continuity of electric and magnetic fields yield such a restriction that $R + \dot{L}$ is always positive. In order to answer this question, we studied Eq. (13) with $u(\chi) = 0$:

$$S(\chi) = \lambda \Phi(\chi) - \chi \frac{\partial \Phi}{\partial \chi} \tag{33}$$

for various values $\nu(\chi)$. We analyzed propagation of a conductivity wave in a system whereby

$$\sigma(\chi) = \sigma_1 \theta(1 - \chi) + \sigma_2 [\theta(\chi - 1) - \theta(\chi - p)], \tag{34}$$

where σ_1 and σ_2 are electric conductivities in regions with $r < \bar{r}_1(t)$ and $\bar{r}_1(t) < r < \bar{r}_2(t)$, respectively, $\theta(\chi)$ is Heaviside function, and in order to provide the existence of the self-similar solution it was assumed that $\bar{r}_2(t)/\bar{r}_1(t) = p = \text{const}$, i.e., the velocity ratio of the trailing edge and front of the conductivity jump remains constant. Without presenting the details of the analysis, just note that for finite p, Eq. (13) with function (34), does not have solutions $\lambda < 0$ and $\nu_0 < 0$ at least in the region with $\text{Im}\lambda = 0$.

Nevertheless, in spite of this negative result we cannot claim that electric current instability cannot occur in a medium without a hydrodynamic flow because the self-similar form of the solution is quite a strong restriction. Moreover, using the magnetostatic and lumped parameter approxima-tions, we showed the feasibility of an electric current dynamo in Ref. [5]. Without going into details, note only that the criterion for excitation of the instability corresponds to the range of parameters where effects of magnetic diffusion cannot be neglected. Thus, currently we cannot answer questions about the feasibility of an electric current dynamo in a medium without a hydrodynamic flow either positively or negatively.

V. CONCLUSIONS

The main goal of this study was to analyze the feasibility of the magnetic dynamo and the electric current dynamo in finite volume systems with a moving front of electric conductivity. Since such a magnetic dynamo requires relatively low values of magnetic Reynolds numbers $\text{Re}_m = (4\pi\sigma/c^2)\bar{r}(t)\dot{\bar{r}}(t) \geq 1$, it is conceivable to suggest that it can occur in various systems. Some of these systems were discussed in our previous studies. Thus in Ref. [1] we analyzed the excitation of the electric current in rapidly expanding cylindrical conductor. In Ref. [5] we investigated an excitation of the electric current in electric circuits with conductors connected in parallel when the electric conductivity of one of the conductors varies rapidly. Here we estimate a possible contribution of the averaged radial motion of the ionosphere to the variation of the magnetic field of the Earth. Although the geometry of this problem (a multilayered sphere) is different from that analyzed in this study, the physics of the phenomenon is essentially the same. Consider the radial expansion of a sphere with a radius $R \sim 10^9$ cm and an electric conductivity $\sigma \sim 10^9$ s^{-1}. Then, the radial velocity which is required for increasing the magnetic field inside this sphere is relatively low, $v \geq c^2/4\pi\sigma R \approx 1$ m/s. Certainly, the excitation of the magnetic dynamo by the ionosphere motions and their effect on the magnetic field of the Earth is a subject of a separate investigation. However, based on the obtained results, it is conceivable to suggest the feasibility of such effects. It is worthwhile noting that the magnetic dynamo considered in this study may also be of relevance in astrophysics in investigations of the role of magnetic fields in stellar evolution and pulsating stars.

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